RECRUITING FOR RENEWAL? SOCIAL CONTROLS AND SPACES IN THE APPRENTICESHIP TO MATHEMATICS

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This study is concerned with participation in mathematics. What do tertiary mathematics students justifications and concerns about the discipline reflect about the apprenticeship to this practice? I have developed a typology, taken from socio-economics, to incorporate methodologies which extend the focus from individuals to the institutional practice.

Questions about socialisation to the discipline

This is a reflection on recruitment to the profession of mathematician made with the intention of investigating the issue of participation in mathematical thinking beyond school. The socialisation process in school mathematics is successful to the extent that many qualify in mathematics. There remains the problem of transfer of the mathematical skill to everyday applications. Even more of an educational challenge, is the difficulty that people experience in taking part in the organization of our society which is increasingly codified in mathematical ways.

The study has focused on the ways those students who have opted to study higher mathematics, relate to mathematical knowledge. In this paper I describe the way I came to look at the socialisation process. The formal access processes are well documented in course work, but do not tell the story of students' accommodation of the institutional demands place upon them as they negotiate the course, nor how they recognize what is required where these are not obvious. I chose to look for the tensions in that process, which were evident in students' talk about the justifications of their decisions and their concerns (Becker, Geer, Hughes and Strauss, 1961). Might this reveal differences in the women's thinking about mathematics, and would these explain the low rates of women's employment in this academic field?

Participants

The data discussed here was taken from interviews conducted with Third Year Undergraduates (NUInt = 36) and Postgraduates (NPInt = 21) in the mathematics department of a large Australian university. These students were amongst the 89 who had returned questionnaires asking them about their relationship with mathematics, mathematicians and their understandings of the nature of mathematics. The questionnaire was distributed to the whole cohort of Third Year Undergraduates (120) and Postgraduates (72).

Dualist thinking

When designing the questionnaire protocol, I had made use of the label, Dualist, a position that Perry had used in his scheme of tertiary Humanities students' intellectual and ethical development (1970). This was the Dualist thinker who felt that the world could be ordered into right and wrong, and that this was done by an authority external to the individual. It was Authority's business to solve any 'grey' areas into these two categories of right and wrong information. Much mathematical activity is taught in this way, and is associated with an external authority, not necessarily distinguished as the institutions of mathematics by school students. The mathematical knowledge that these institutions have produced is based on the sorts of logical thinking, that itself is inspired by the philosophical position of Dualism. This attributes to the knowledge production, a sort of neutrality, associated with the Mind-Body split, Enlightenment ideals and a Boubakian project to develop the discipline in increasingly abstract ways (Heelan, 1975, Struik, 1981, Gleick, 1987).

Mathematical ways of working, at least in parts of the secondary school curriculum, are characterised by the 'certainties' of mathematics, and indisputable 'facts' (Webber, 1998). There is, generally, little discussion of the possibilities of its uncertainties, perhaps because of time constraints (Skovsmose, 1994). One of the criteria for socialisation into mathematics may be a willingness to accept the 'truths' of the discipline, and to understand later. The interesting question is to know what currently inspires the projects of mathematics, how the institution sustains its authority and how the recruitment process is used to in these.

Protocols

In the questionnaire, I asked a question about 'the certainties of mathematics'. The intention was to signal to students the issue of the reliability of mathematical knowledge, so that those who volunteered to come to an interview, might consider the part that these certainties had played in attracting them to a study of mathematics. I hoped to learn what forms this Dualist thinking took, and how this fitted with the notion of creative thinking and renewal in mathematics. It seemed at once, both essential to mathematical thinking and yet counter-productive to its development. It also seemed that to become a mathematician, it was essential to show one's identification with this way of thinking.

Initial findings

There were few differences in the questionnaire response rates between the men and women at the undergraduate level. I had asked participants to prepare for the interview by thinking of five or six people who had opened up mathematics for them. I was struck by their enthusiasm for the subject, and admiration of lecturers. A count of who these mediators of mathematics were, revealed that Postgraduates (96%) were twice as likely to name a family member as Undergraduates (48%).

Students' relationships with mathematical knowledge forms

STUDENTS' IDENTIFICATION WITH DISCIPLINARY ACTIVITY

Students showed an appreciation of the *organization and resilience of mathematical knowledge*, which distinguished it from other forms of knowledge.

> It was very neat and logical.
> [I like] the fact that its results could not be questioned.
> I like maths because it forms a rigorous basis [for thinking].
> I enjoyed its certainty and rigidity as opposed to say, arts subjects.
> [I liked] the clear way in which it fitted together logically.
> [I liked] the problem solving and rigour.
> [I liked] the disciplined image.

There are several dimensions to the spaces opened up in this mathematical activity. The following excerpts from undergraduate students show how it extends from the personal satisfaction of meeting course requirements, to a way of seeing the world through mathematical lenses. At every point there appears to be a pleasure in the achievement of procedural competence, and delight in being able to act effectively on problems with mathematical tools. Most Undergraduates talked enthusiastically about what could be done with a mathematical orientation.

> F5: 'Elegance and ease with which you can express yourself.'
>
> F21: '[Mathematics] provided a structured conceptual framework that could be used to explore and understand the real world. The applications of the theorems appealed to me enormously.'
>
> M21: 'The way in which it could be used to explain things. Clear way in which it fitted together logically. I enjoyed the subject, and it was interesting. It had a clear and logical basis. Other subjects were _ are more vague.'
>
> M49: 'It is interesting and useful, and essential to an understanding of physics, which is what I want to do. … The way it explained physical systems.'

Identification of students: F for a female student, and M for a male student followed by their respective number.

They remarked on the nature of working mathematically. These comments point to the challenge and scope students felt rather than the constraints of the discipline.

F4: working mathematically was like
'[working with] crossword puzzle, [being a] detective; here are the clues. What is the answer, [or at least] an answer.'

F10: 'Something challenging; freedom to explore different possibilities.'

F19: 'It's good to gain an understanding of nature. It's also fun. A bit of fun; [I] like the doing of it, even Pure.'

M50: 'Pure, logical, interesting. Good at it; liked it.'

Many, when asked if they liked 'the certainties of mathematics' in questionnaires, responded that they did (Undergraduates: 46%; Postgraduates even more strongly: 58%). One dimension of that certainty is presented by M17.

M17: 'It appeared to have meaning rather than hypothetical assumptions. I always knew where I was at with it. Related to it better than the biological side of science.'

The implication appears to be that meanings are more stable in mathematics and that unlike much scientific thinking, it was not working with hypotheses in the same way.

M26: It was less confusing than physics. It was the least boring.

M31: '[I like] the problem solving and rigour.'

Students were interested in the bounded nature of mathematical space, as a framework within which to work. Their conceptions of this framework was not simply as a set of guidelines.

F7: 'I was very good at it. It was very neat and logical. I enjoyed doing it.'

Lecturers had addressed the issue of 'certainty' at the beginning of the Undergraduate course, and appeared to have reinforced the authority of the mathematical solution, when they communicated their own conviction about the validity of the mathematics, to this student.

F14: 'I liked the first year courses since they explained the reason behind all the maths you so far took as "truths". … Challenging to get an answer, and then [be] able to know it was right.'

RECOGNIZING STUDENTS' COUNTER-NORMATIVE POSITIONS

These are socialised views, conforming to the projects of mathematics as they are widely conceived, rather than questioning them. Was there evidence of a counter-normative stance, one communicating the change processes that are under way in that department? What indications do students have of the thinking of mathematicians at a time when knowledge productions are undergoing great changes (Gibbons, Limoges, Nowotny, Schwartzman, Scott, and Trow, 1994)? Is mathematics immune from all this change? The historiographers of mathematics (Mehrtens, Bos, and Schneider, 1981) and sociologists of scientific knowledge (Barnes, Bloor, and Henry, 1996; Kuhn, 1970; Restivo, 1992) believe that it is not. What challenges are there to the existing patterns of normalisation from students? Do those students who question drop out earlier in the process, or learn to withhold their opinions in the processes of recruitment, while sustaining personal beliefs about the possibilities that mathematics offers?

In the following examples of two women's approaches to the course, it is apparent that my earlier plan just to explore the relationship between individuals and the knowledge forms of mathematics, in isolation from its institutional contexts, was inadequate. In the second example, it is apparent that for a student to dwell only on the aesthetics of mathematics, however inspirational these were, and to neglect the dynamic of the political activity in its institutions was a path to isolation rather than membership of the profession.

These two Undergraduates women who appeared to be equally strongly drawn to mathematics, provided an insight to the difficulties of identifying what is needed to become a mathematician. They presented a sharp contrast in approach to accessing the institution.

Sophia had set about learning and demonstrating a technical competence, which included being able to manipulate three-dimensional shapes in her head, and learning to use the specific terminologies associated with particular topics. She tended to work alone, but to ask questions of tutors and, when more sure of her understanding, of lecturers too. She worked with the idea that she should demonstrate her competence. Learning how to do this, she believed, was a key part of her survival in the apprenticeship.

Myra was engaged in working intuitively and in producing her own solutions. Her debate about mathematics was with friends rather than lecturers in the department, and she was perhaps more attracted to the beauty of the solutions than many of the Undergraduates, who seemed more dependent on lecturers. She was conscious of working differently, in what she called, 'unconventional ways.' She had nevertheless been shocked at having an assignment, using own solutions rejected, without comment from the lecturer, apart from a series of zeros on the work.

This was clearly an instance of reining in a student's activity, and probably disqualifying her from any chance of a scholarship to study in the Fourth Year. The space in which she could explore mathematics was limited by this 'economy' in the institutional practice. It could be justified as weeding out those who do not conform.

Her 'unconventional' approach invites two distinct responses from a researcher. One comes from a psychological position, which treats non-conformity as deviant. If her behaviour is viewed in this Durkheimian sense, her behaviour is a pathological one, to be corrected. The other is a sociological one, which considers that there is conflict in social processes, and that resistance is a component part of these processes. I needed to extend the initial methodological approach to see how individuals' activity has been interpreted sociologically.

Extending the methodological approach

I made use of a typology, taken from socio-economics, devised by Boudon and Bourricaud (1989) to extend methodologies used in school education to incorporate those that deal more appropriately with negotiation of an apprenticeship.

Typology

	Individualistic	**Holistic**
'Rational'	*Type 1* Educational psychology: performance criteria established	*Type 2* Sociological (structuralist) and scientific (positivist) accounts
'Irrational'	*Type 3* Grounded studies: accountable to participants	*Type 4* Deconstructionist accounts Reflective practice

The methodologies grouped on the left hand side of the table, in Types 1 and 3, focus on the individual. Individualistic methodologies tend to attribute agency to individuals, and to assume that the student has the option to participate, as from a more or less level playing field. In between the left and right hand side of the table are those methodologies, which situate the individual within a cultural space, in which participation is regulated by the institution. Those on the right hand side focus on the social or holistic aspects social activity. A study of the controls of the institutions of mathematics belongs on this side.

Type 2, structural accounts of institutional reproduction (Melucci, 1997) and the maintenance of knowledge productions (von Cranach, 1992), although they signal the ways the interests of the institutions are served, only partly explain what happens in a specific apprenticeship. In order to bring together and make visible, both the students' activity and the institution's activity in this apprenticeship, I have drawn on both individualistic and holistic methodologies, and situate the study in the mid-space, asking questions from each.

What were the openings that students saw in mathematics? What could a researcher deduce about the accessible spaces of working mathematically at undergraduate and postgraduate level? What was their relationship with institutional controls? And what could be learnt about the nature of socialisation to this profession?

The attribution of the terms Rational and Irrational is arbitrary. Rational activity in becoming a mathematician may look more like Sophia's than Myra's. It is difficult for an outsider to recognize what is the most suitable behaviour to gain access in specific circumstances. There is not one right conception of what it means to become a mathematician. Practitioners within it, take on multiple roles, but membership remains only partial (Lave and Wenger, 1991).

The tension between learning for examinations and having an interest in the mathematics is clear in the explanations of students. What is required of students is clearly linked to the credentialling function of the institution, but the process of becoming a mathematician is less clear.

Undergraduates repeatedly named two characteristics that one had to have to become a mathematician. One was *intelligence* and the other was *interest in mathematics*. The paths to these states remained unexplained. The path to success, they implied, were often contradictory. That you had to *work hard*, *steadily*, *and make an effort to understand* what you were listening to, was not in question. However, if you spent too long trying to understand some concept, you might not leave time to learn other important material for the examinations. Several students commented on the department's wastefulness in not acknowledging the ability in other students who had helped them to understand. They believed these had a better understanding than they themselves had, but had somehow not performed as well in the examinations as they had. They remarked on their ability to *learn it all up in three days before the exam*. They knew that what they had learned would quickly be forgotten, unless they could revive it in a later unit of study.

Although Postgraduate women intended to do postdoctoral studies at the same rates as the men, their positive responses to the questionnaire item:

> *I would like to become a mathematician*

were much lower. At Undergraduate level, they had been similar. In order to understand better what is happening in this recruitment process with respect to the low rates of employment of women, I have extended my questions to ask, how does this institution appear to make use of recruits in its renewal process?

Recruitment to projects of mathematics

And how well does the institution serve mathematics in an age of modernity, in which some aspects of disciplinary activity have been challenged (Fraser and Nicholson, 1994, Code, 1993, Addelson, 1993, Dant, 1991, Marcuse, 1968), rather than projects of an earlier age (Polanyi, 1968, Barrow, 1988, Penrose, 1994)? What evidence is there of reflective practice (Bourdieu and Wacquant, 1992) bringing into the arena elements of counter-normative thinking, and sharing these with students?

The question framed in holistic terms requires that consciousness about these projects, their worth and ways of working, be raised. What types of consciousness within the practice of mathematics, are shared with recruits, and what spaces are made accessible in this way? The awareness and what students make of it, do not depend solely on the acquisition of course material, but are held in the wider and immediate cultural community as values and beliefs about the possibilities of and for mathematics.

Many students in the Third Year of the undergraduate course, gave as one of their reasons for continuing in their study of mathematics, a preference for working on problems within a framework. They gave accounts that resonate with popular understandings of what it means to work mathematically, which in themselves merely confirm an image of what it means to *be* mathematical. But when these accounts are put together, and analysed for the characteristics of that *mathematical space*, it is possible to see what types of thinking are valued by students and, indirectly, the mathematical community. They constitute a first hand account of those institutional controls by people considering membership of that institution.

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